

Constraint on $B - L$ cosmic string from leptogenesis with degenerate neutrinos

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Abstract

In the early Universe, as a consequence of $U(1)_{B-L}$ gauge symmetry-breaking, the so-called $B-L$ cosmic strings are expected to be produced at the breaking scale η_{B-L} according to the Kibble mechanism. The decaying, collapsing closed loops of these strings can release the right-handed neutrinos, whose subsequent decay can contribute to the baryon asymmetry of the Universe (BAU), through the "slow death"(SD) process and/or the "quick death"(QD) process. In this paper, we assume that the decay of the lightest heavy Majorana neutrinos released from the $B-L$ cosmic string loops can produce a baryon asymmetry consistent with the cosmic microwave background (CMB) observations. Considering the fact that both the neutrinoless double beta decay experiment and the cosmological data show a preference for degenerate neutrinos, we give the lower limits for the breaking scale η_{B-L} with the neutrino masses $0.06\text{eV} \leq \bar{m} = (m_1^2 + m_2^2 + m_3^2)^{1/2} \leq 1.0\text{eV}$, where the full possible cases of degenerate neutrinos are included. We obtain $\eta_{B-L} \gtrsim 3.3 \times 10^{15}\text{GeV}$, $5.3 \times 10^{15}\text{GeV}$ and $9.5 \times 10^{15}\text{GeV}$ for $\bar{m} = 0.2\text{eV}$, 0.4eV and 1.0eV respectively in the SD process, and find the $B-L$ cosmic string has a very small contribution to the BAU in the QD process.

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The baryon asymmetry of the Universe (BAU) has been determined precisely[1]:

$$\eta_B^{CMB} \equiv \frac{n_B}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10}, \quad (1)$$

where $n_B = n_b - n_{\bar{b}}$ and n_γ are the baryon and photon number densities, respectively. At the same time, there are several neutrino oscillation experiments[2, 3] which have confirmed the extremely small but non-zero neutrino masses. Then leptogenesis[4] is now an attractive scenario which can simultaneously explain the cosmological baryon asymmetry and the neutrino properties by the seesaw mechanism[5].

The simplest leptogenesis scenario is to extend the standard model (SM) by three generations of the right-handed neutrinos with Majorana mass. A more appealing alternative is to consider this within the context of unified models with an embedded $U(1)_{B-L}$ gauge symmetry, which can be derived from the $SO(10)$ models. After spontaneous breaking of the $U(1)_{B-L}$ gauge symmetry, the right-handed neutrinos naturally acquire heavy Majorana mass and produce a lepton asymmetry, which finally converted to the baryon asymmetry via the $(B-L)$ -conserving sphaleron process[6], by decaying into massless leptons and electroweak Higgs bosons.

In the early Universe, according to the Kibble mechanism[7], the so-called $B-L$ cosmic strings are expected to be produced at the $U(1)_{B-L}$ symmetry-breaking scale during $SO(10)$ breaking to the SM gauge group[8]. The strings are formed by the gauge field and Higgs field, and the Higgs field also gives heavy Majorana mass to the right-handed neutrinos through Yukawa coupling[9, 10]. As discussed in Ref.[11, 12, 13, 14, 15, 16], the decaying, collapsing closed loops of these strings can be a nonthermal source of the right-handed neutrinos whose subsequent decay can contribute to the BAU.

When the $B-L$ cosmic string loops contribute significantly to the BAU, ones find that the $U(1)_{B-L}$ gauge symmetry-breaking scale η_{B-L} has a lower limit $\eta_{B-L} \gtrsim 1.7 \times 10^{11} \text{GeV}$ [16] if the light neutrino masses are hierarchical, especially for $m_3 \simeq 0.05 \text{eV}$, where $m_i (i = 1, 2, 3)$ is the eigenvalue of the light neutrino mass matrix. But once the evidence of neutrinoless double beta decay with $m_{ee} = (0.05 - 0.86) \text{eV}$ (at 95 % C.L.)[17] is confirmed, a degenerate neutrino spectrum is required. Recent studies on the cosmological data[1, 18] with the neutrino oscillation experiment results[2, 3] also showed a preference for degenerate neutrinos with $m_i \lesssim 0.23 \text{eV}$ or $m_i \lesssim 0.56 \text{eV}$.

In this paper, we follow the discussions in Refs.[16] and assume that the lightest heavy

Majorana neutrinos are released from the $B - L$ cosmic string loops, and hence a baryon asymmetry consistent with the observations can be induced by their decay modes. We estimate the $U(1)_{B-L}$ symmetry-breaking scale η_{B-L} for $0.06\text{eV} \leq \bar{m} \leq 1.0\text{eV}$, where \bar{m} is defined as $\bar{m}^2 = m_1^2 + m_2^2 + m_3^2$. In this range of \bar{m} , the full possible cases of degenerate neutrinos are included.

In the generic picture, local $B - L$ cosmic strings form at the phase transition associated with the spontaneous symmetry breaking of $U(1)_{B-L}$. During the phase transition, a network of strings forms, consisting of both infinite strings and cosmic loops. After the transition, the infinite string network coarsens and more loops form from the intercommuting of infinite strings. In the following discussion, we pay particular attention to the formation of closed loops and their subsequent evolution.

The formation and evolution of the cosmic string loops have been studied extensively, both in analytical and numerical methods, for details see [19, 20]. After their formation, the evolution of the closed loops can be broadly categorized into two classes. One is the "slow death" (SD) process [14, 19, 20], where the loops born at time t have a longer lifetime (compared to the Hubble expansion time scale $H^{-1}(t)$). In this scenario, during the phase transition, the formed string loops oscillate freely and lose their energy by emitting gravitational radiation. When the loop's radius becomes of the order of the string width, the loop releases its final energy into massive particles. Among these particles will be the massive gauge bosons, Higgs bosons and massive right-handed neutrinos which were trapped in the string as fermion zero modes [19]. The other is the "quick death" (QD) process [21]—the loops die instantaneously as soon as they are formed due to the high probability of self intersecting [22]. Thus they would lose only a negligible amount of energy in gravitational radiation and massive particle radiation rather than gravitational radiation plays the dominant role.

For the above two cases, the number density of loops disappearing in the radiation dominated epoch at any time t can be described respectively as [16, 19, 20]:

$$\frac{dn_{SD}}{dt} = f_{SD} \frac{1}{x^2} (\Gamma G \mu)^{-1} \frac{(C+1)^{3/2}}{C} t^{-4}, \quad (2)$$

$$\frac{dn_{QD}}{dt} = f_{QD} \frac{1}{x^2} \mu^{1/2} t^{-3}, \quad (3)$$

where x is approximately in the range $\sim 0.4 - 0.7$ supported by the extensive numerical simulations, $\Gamma \sim 100$ is a geometrical factor that determines the average loop length, μ is

the mass per unit length of a cosmic string and related the symmetry-breaking scale η with $\mu^{1/2} \sim \eta$, C is a numerical factor of order unity and $G = 1/M_{Pl}^2$ is the Newton's constant, while f_{SD} and f_{QD} denote the fraction of newly born loops which die through the SD process and QD process respectively.

Noteworthy that the observations of the cosmic microwave background (CMB) anisotropy give an upper bound on the symmetry-breaking scale[23]

$$\eta \lesssim 1.0 \times 10^{16} GeV. \quad (4)$$

In addition, the measured flux of the cosmic gamma ray background in the 10MeV – 100GeV[24] energy region puts a constraint on f_{QD} [25]

$$f_{QD}(\eta/10^{16} GeV)^2 \lesssim 9.6 \times 10^{-6}, \quad (5)$$

but there is no equivalent constraint on f_{SD} . This difference can be understood easily, since the time dependence of the disappearing rate of loops is $\propto t^{-4}$ in the SD case [see Eq.(2)] while $\propto t^{-3}$ in the QD case [see Eq.(3)], in other words, the SD process dominates at sufficiently early time, while the QD process dominates at relatively late time and can potentially contribute to the nonthermal gamma ray background.

It is difficult to calculate exactly the total number of the heavy Majorana neutrinos from each loop, but as shown in Ref.[14] when a cosmic loop decays, it releases at least one heavy Majorana neutrino. For simplicity, we may expect that it would be a number of order unity. Then the releasing rate of the heavy Majorana neutrinos $N_i (i = 1, 2, 3)$ from the SD process and QD process at any time t can be written as:

$$\frac{dn_{N_i}^{SD}}{dt} = N_{N_i}^{SD} f_{SD} \frac{1}{x^2} (\Gamma G \mu)^{-1} \frac{(C+1)^{3/2}}{C} t^{-4}, \quad (6)$$

$$\frac{dn_{N_i}^{QD}}{dt} = N_{N_i}^{QD} f_{QD} \frac{1}{x^2} \mu^{1/2} t^{-3}, \quad (7)$$

with $N_{N_i}^{SD}$ and $N_{N_i}^{QD} \sim O(1)$.

In the leptogenesis scenario, the decay of the heavy Majorana neutrinos N_i which produces the lepton asymmetry is described by the following Lagrangian

$$-\mathcal{L} = h_{ij} \bar{l}_{Li} \phi \nu_{Rj} + \frac{1}{2} M_i \bar{\nu}_{Ri}^c \nu_{Ri} + H.c., \quad (8)$$

where l, ϕ are the SM leptons and Higgs doublets respectively. In this framework, the heavy Majorana neutrino is given by $N_i = \nu_{Ri} + \nu_{Ri}^c$ with heavy mass M_i . The light neutrino mass

matrix can be written as

$$m_\nu = -h^* \frac{1}{M} h^\dagger v^2 \quad (9)$$

with $M = \text{diag}(M_1, M_2, M_3)$ and $v = \langle \phi \rangle \simeq 174 \text{ GeV}$.

For a hierarchical spectrum of the heavy Majorana neutrinos $M_1 \ll M_2, M_3$, the lepton asymmetry, which is finally converted to the baryon asymmetry, comes mainly from the decay of the lightest heavy Majorana neutrino N_1 and the contribution of the cosmic string loops to the BAU can be estimated as:

$$\eta_B = \frac{28}{79} \times 7.04 \times \varepsilon_1 \int_{t_F}^{t_{EW}} \frac{1}{s} \left(\frac{dn_{N_1}}{dt} \right) dt, \quad (10)$$

where $28/79$ is the value of $B/(B-L)$ for SM[26], 7.04 is the present density ratio of photon number and entropy, ε_1 is the CP asymmetry of N_1 decays and $s = (2\pi^2/45)g_*T^3$ is the entropy density with $g_* \simeq 106.75$. t_{EW} is the electroweak transition time, while t_F denotes the epoch when the inverse decays and L -nonconserving scatterings begin to freeze out and there is no washout effects any more. $\frac{dn_{N_1}}{dt}$ is the releasing rate of N_1 from the cosmic string loops. The temperature T and time t are related by

$$t = \frac{1}{2H(T)} \quad (11)$$

with Hubble constant

$$H(T) = \sqrt{\frac{8\pi^3 g_*}{90}} T^2 / M_{Pl}. \quad (12)$$

Therefore we get

$$dt = -\frac{1}{TH(T)} dT \quad (13)$$

and can rewrite Eq.(10) as

$$\eta_B = \frac{28}{79} \times 7.04 \times \varepsilon_1 \int_{T_{EW}}^{T_F} \frac{1}{s} \left[\frac{dn_{N_1}}{dt}(T) \right] \frac{1}{TH(T)} dT. \quad (14)$$

Now the key to calculate the baryon asymmetry in Eq.(14) is how to determine ε_1 and T_F . We note that there is an upper bound on ε_1 with $m_1 < m_2 < m_3$ [27] from neutrino oscillation experiments:

$$|\varepsilon_1| \leq \varepsilon_1^{max} = \frac{3}{16\pi} \frac{M_1}{v^2} \frac{\Delta m_{atm}^2 + \Delta m_{sol}^2}{m_3} \simeq 5.27 \times 10^{-18} \frac{(M_1/\text{GeV})}{(m_3/\text{eV})}, \quad (15)$$

where we have used $\Delta m_{atm}^2 = m_3^2 - m_2^2 = 2.6 \times 10^{-3} \text{ eV}^2$ for atmospheric neutrinos[2] and $\Delta m_{sol}^2 = m_2^2 - m_1^2 = 7.1 \times 10^{-5} \text{ eV}^2$ for solar neutrinos[3]. We also calculate T_F for

$$1 \ll K \lesssim 10^6 [28]$$

$$T_F \simeq \frac{M_1}{4.2(\ln K)^{0.6}} \quad (16)$$

with

$$K \equiv \frac{\Gamma_{N_1}}{H(T)}|_{T=M_1}. \quad (17)$$

Here Γ_{N_1} is the decay width of N_1 . Using

$$\Gamma_{N_1} = \frac{1}{8\pi} (h^\dagger h)_{11} M_1, \quad (18)$$

and Eq.(12), we get

$$K = \frac{\tilde{m}_1}{m_*} \quad (19)$$

with

$$\tilde{m}_1 = \frac{(h^\dagger h)_{11} v^2}{M_1}, \quad (20)$$

$$m_* = \frac{16\pi^{5/2} g_*^{1/2}}{3\sqrt{5}} \frac{v^2}{M_{Pl}} \simeq 1.07 \times 10^{-3} eV. \quad (21)$$

Since $\tilde{m}_1 \geq m_1$ [29], we replace \tilde{m}_1 by m_1 in Eq.(19) and then give a lower limit for K

$$K \geq K_{min} \equiv \frac{m_1}{m_*}, \quad (22)$$

accordingly T_F has an upper bound

$$T_F \leq T_F^{max} \simeq \frac{M_1}{4.2(\ln K_{min})^{0.6}}. \quad (23)$$

Assuming that the decay of the lightest heavy Majorana neutrinos, which are released from the cosmic string loops, can produce a baryon asymmetry consistent with the CMB observations (1), we obtain the following restriction

$$\eta_B(\varepsilon_1^{max}, T_F^{max}) = \frac{28}{79} \times 7.04 \times \varepsilon_1^{max} \int_{T_{EW}}^{T_F^{max}} \frac{1}{s} \left[\frac{dn_{N_1}}{dt}(T) \right] \frac{1}{TH(T)} dT \geq \eta_B^{CMB}. \quad (24)$$

Then the $U(1)_{B-L}$ gauge symmetry-breaking scale can be estimated by using the above equation in the SD process and QD process respectively. In the following calculations, we will take $x = 0.5$, $\Gamma = 100$, $C = 1$, $\mu = \eta_{B-L}^2$, and $M_1 = g_1 \eta_{B-L}$, where the Yukawa coupling $g_1 \lesssim 1$ is natural for $M_1 \ll M_2, M_3$.

In the SD case, using Eq.(6), (11), (14), (15) and (23), we obtain

$$\eta_B^{SD}(\varepsilon_1^{max}, T_F^{max}) = \frac{28}{79} \times 7.04 \times \varepsilon_1^{max} \int_{T_{EW}}^{T_F^{max}} \frac{1}{s} \left[\frac{dn_{N_1}^{SD}}{dt}(T) \right] \frac{1}{TH(T)} dT$$

$$\begin{aligned}
&\simeq \frac{28}{79} \times 7.04 \times \varepsilon_1^{max} \times 5.33 \times 10^{-18} N_{N_1}^{SD} f_{SD} \frac{(T_F^{max}/GeV)^3 - (T_{EW}/GeV)^3}{(\eta_{B-L}^{SD}/GeV)^2} \\
&\simeq 9.44 \times 10^{-37} g_1^4 N_{N_1}^{SD} f_{SD} \frac{(\eta_{B-L}^{SD}/GeV)^2}{(m_3/eV)[\ln(m_1/m_*)]^{1.8}}.
\end{aligned} \tag{25}$$

In the last step we have neglected the effect of $T_{EW} \simeq 100\text{GeV}$ since the dominant contribution to the integral comes from $T_F \gg T_{EW}$. Considering the constraint (24) from the CMB, we can get a lower limit for η_{B-L}

$$\eta_{B-L} \gtrsim 2.39 \times 10^{13} \frac{1}{g_1^2 (N_{N_1}^{SD})^{1/2} f_{SD}^{1/2}} (m_3/eV)^{1/2} [\ln(m_1/m_*)]^{0.9} GeV \tag{26}$$

where the 3σ lower limit for $(\eta_B^{CMB})_{low} = 5.4 \times 10^{-10}$ has been adopted. Using the relations

$$m_1^2 = \frac{1}{3}(\bar{m}^2 - \Delta m_{atm}^2 - 2\Delta m_{sol}^2), \tag{27}$$

$$m_3^2 = \frac{1}{3}(\bar{m}^2 + 2\Delta m_{atm}^2 - \Delta m_{sol}^2), \tag{28}$$

and fixing $N_{N_1}^{SD}$ and f_{SD} , we can get the lower limits for η_{B-L} with \bar{m} .

In Fig.1 we show η_{B-L} as a function of \bar{m} for $0.06\text{eV} \leq \bar{m} \leq 1.0\text{eV}$ (corresponding to $10 \lesssim k \lesssim 300$) with $g_1 = 1.0, 0.1, 0.01$ by taking $N_{N_1}^{SD} = 1$, $f_{SD} = 1$. We find that in order to satisfy the CMB constraint $\eta_{B-L} \lesssim 1.0 \times 10^{16}\text{GeV}$, the Yukawa coupling $g_1 \gtrsim 0.1$. In the case of degenerate neutrino scenario with $g_1 = 0.1$, we obtain $\eta_{B-L} \gtrsim 3.3 \times 10^{15}\text{GeV}$ for $\bar{m} = 0.2\text{eV}$ (corresponding to the upper bound for the successful leptogenesis[29]), and $\eta_{B-L} \gtrsim 5.3 \times 10^{15}\text{GeV}$ for $\bar{m} = 0.4\text{eV}$ or $\eta_{B-L} \gtrsim 9.5 \times 10^{15}\text{GeV}$ for $\bar{m} = 1.0\text{eV}$ (corresponding to the upper bounds from the cosmological data[1, 18]).

Replacing $\frac{dn_{N_1}}{dt}$ in Eq.(14) by $\frac{dn_{N_1}^{QD}}{dt}$ given by Eq.(7), considering the additional constraint on f_{QD} from Eq.(5) and repeating the same steps in the SD case above, we can also obtain the baryon asymmetry and the lower limit for η_{B-L} in the QD process

$$\eta_B^{QD}(\varepsilon_1^{max}, T_F^{max}) \simeq 4.05 \times 10^{-27} g_1^2 N_{N_1}^{QD} \frac{1}{(m_3/eV)[\ln(m_1/m_*)]^{0.6}} (\eta_{B-L}/GeV), \tag{29}$$

$$\eta_{B-L} \gtrsim 1.33 \times 10^{17} \frac{1}{g_1^2 N_{N_1}^{QD}} (m_3/eV) [\ln(m_1/m_*)]^{0.6} GeV. \tag{30}$$

In Fig.2 we plot η_{B-L} as a function of \bar{m} for $g_1 = 1.0, 0.1, 0.01$ by taking the parameters as $N_{N_1}^{QD} = 1$. We find that the lower limits for η_{B-L} with $0.06\text{eV} \leq \bar{m} \leq 1.0\text{eV}$ in the QD process are much higher than the values in the SD process, and higher than the upper bound $1.0 \times 10^{16}\text{GeV}$. Furthermore by taking $\eta_{B-L} = 1.0 \times 10^{16}\text{GeV}$ and $N_{N_1}^{QD} = 1$ in Eq.(29), we

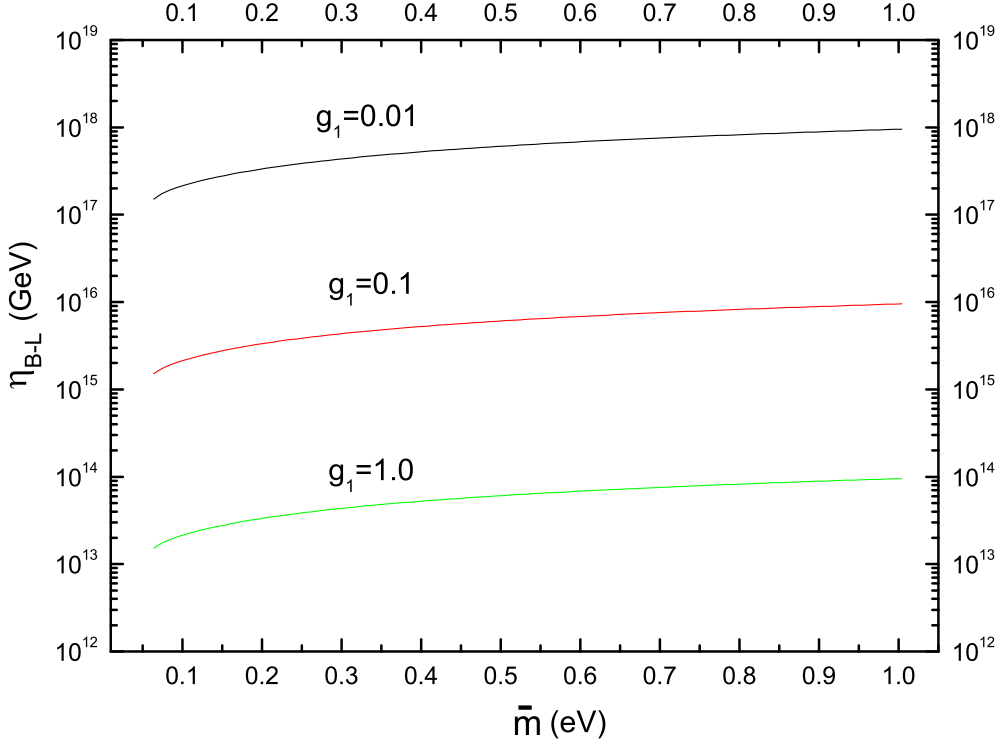


FIG. 1: The evolution of the lower limits for η_{B-L} with $0.064\text{eV} \leq \bar{m} \leq 1.0\text{eV}$ in the SD process for $N_{N_1}^{SD} = 1$, $f_{SD} = 1$.

plot the evolution of η_B^{QD} as a function of \bar{m} in Fig.3. We can see that the contribution from the $B - L$ cosmic string loops to the BAU is small enough to be neglected with the above neutrino masses for the SD case.

In summary, we study such leptogenesis scenario: the lightest heavy Majorana neutrinos are released from the $B - L$ cosmic string loops, and their decay can produce a baryon asymmetry consistent with the CMB observations. Considering the fact that both the neutrinoless double beta decay experiment and the cosmological data show a preference for the degenerate neutrinos, we give the lower limits for the $U(1)_{B-L}$ symmetry-breaking scale η_{B-L} with $0.06\text{eV} \leq \bar{m} \leq 1.0\text{eV}$, where the full possible cases of degenerate neutrinos are included. Especially we plot the lower limits for η_{B-L} with \bar{m} in the SD process and QD process respectively.

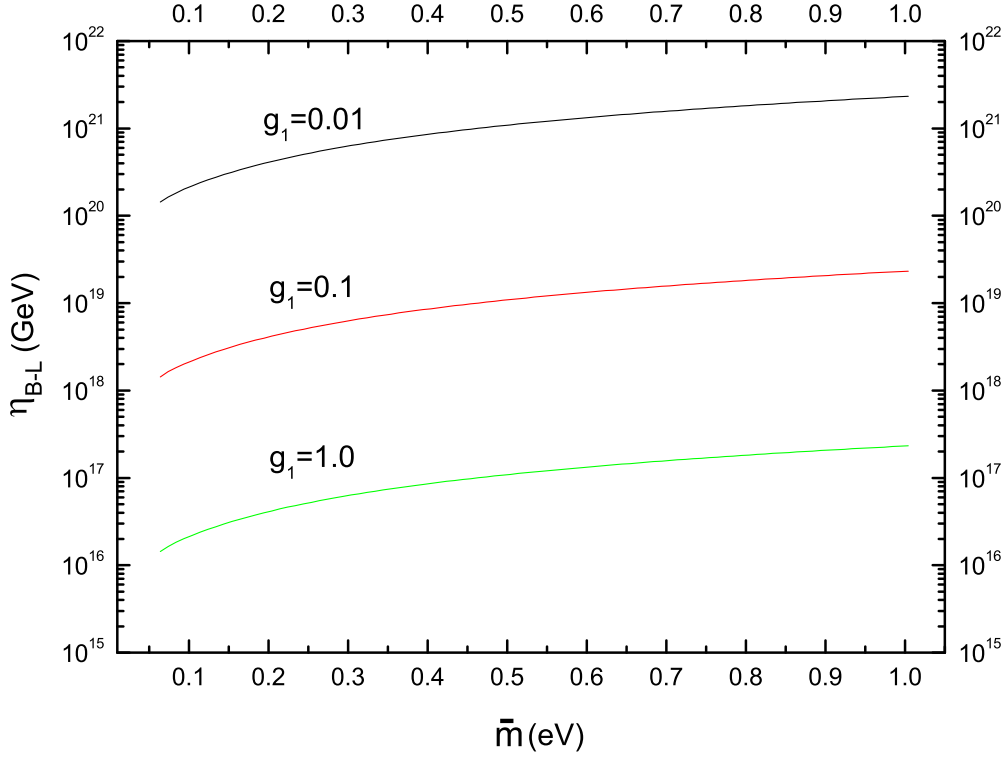


FIG. 2: The evolution of the lower limits for η_{B-L} with $0.064\text{eV} \leq \bar{m} \leq 1.0\text{eV}$ in the QD process for $N_{N_1}^{QD} = 1$.

In the SD process, we find that the Yukawa coupling g_1 should be $\gtrsim 0.1$ due to the CMB constraint $\eta_{B-L} \lesssim 1.0 \times 10^{16}\text{GeV}$. In the case of degenerate neutrino scenario, we obtain $\eta_{B-L} \gtrsim 3.3 \times 10^{15}\text{GeV}$, $5.3 \times 10^{15}\text{GeV}$ and $9.5 \times 10^{15}\text{GeV}$ for the degenerate neutrino masses $\bar{m} = 0.2\text{eV}$, 0.4eV and 1.0eV , respectively. And we also find that there is very small contributions from the $B - L$ cosmic strings to the BAU in the QD process.

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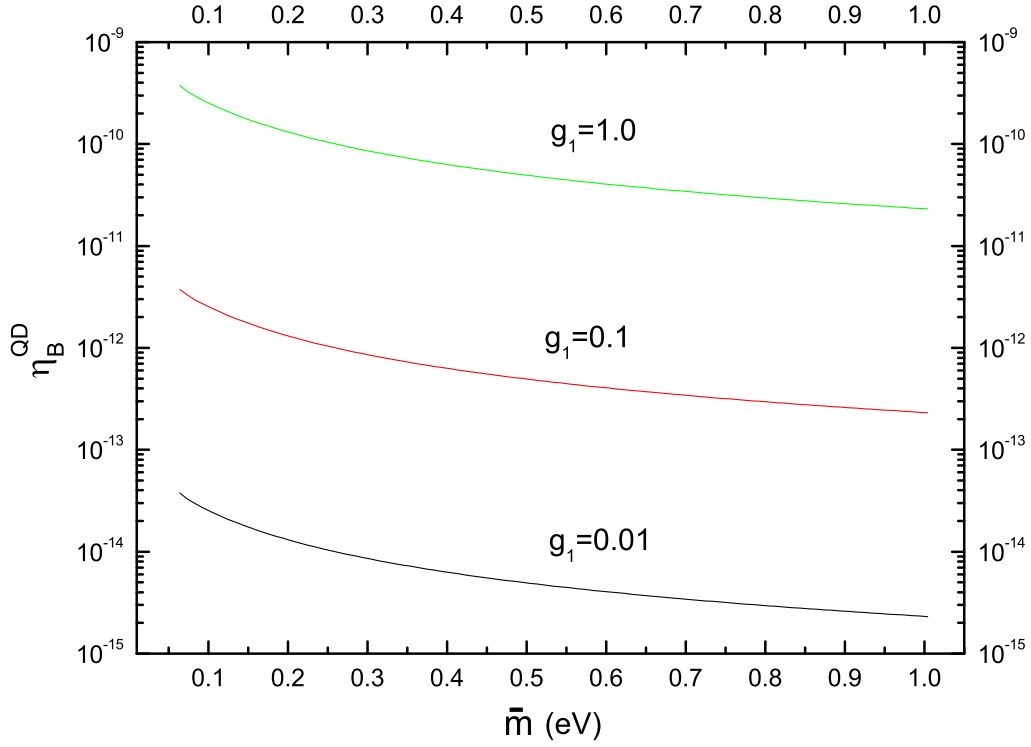


FIG. 3: The evolution of η_B^{QD} with $0.064\text{eV} \leq \bar{m} \leq 1.0\text{eV}$ in the QD process for $N_{N_1}^{QD} = 1$ and $\eta_{B-L} = 1.0 \times 10^{16}\text{GeV}$.

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